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## FAILURE MECHANICS OF COHESIVE-FRICTIONAL MATERIALS

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**Executive Summary**

In the course of this project analytical methods were developed to study the formation of discontinuous failure modes in softening and non-associated pressure-sensitive elastoplasticity. The analytical solutions were interpreted geometrically in the form of Mohr-type envelope concepts to detect the onset of localization. The theoretical failure modes were validated with the help of computational simulations on a representative volume element. The main objective was to assess the regularization properties of Cosserat continua which feature an internal length scale in addition to non-symmetric stress and strain tensors because of couple-stresses and micro-rotations as compared to classical Boltzmann continua. To this end the strength concept of Mohr was extended to non-symmetric stress states to analyze localization of discontinuous failure. The Mohr representation of stress led to a geometrical condition which was used to determine the critical values of hardening/softening and the localization directions that characterize the failure plane and the failure direction. This geometric interpretation of the localization condition led to a systematic study of spatial discontinuities and how they were regularized in elastoplastic Cosserat continua as compared to non-polar media.

For illustration, the theoretical findings were augmented by extensive computational failure studies that were carried out on a representative volume element made of pressure-sensitive elastoplastic Cosserat material of the Drucker-Prager type. The computational failure simulations did demonstrate the non-local character of Cosserat continua that was depending on the degree of non-symmetry of stress and strain as well as on the internal length scale of the micropolar medium. The computational results did illustrate the conversion of discontinuous failure from mixed-mode failure to mode I decohesion and separation, irrespectively of the far-field stresses. In fact, this conversion of discontinuous failure modes was most notable when localization was studied asymptotically at the border between polar and non-polar behavior.

During the course of this project Ms. Maria-Magdalena Iordache completed her Ph.D. dissertation on '*Failure Analysis of Classical and Micropolar Elastoplastic Materials*', which did result in two major journal publications and several conference papers listed at the end of this report.

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## Introductory Remarks

Aside from stiffness and strength, it is mainly the ductility that distinguishes between the brittle and ductile appearance of failure in components and structures. In view of the increasing use of high performance and non-conventional concrete materials in construction, the concomitant decrease of ductility is of paramount importance for the integrity of structures subject to static and dynamic loading.

Cohesive-frictional materials, such as concrete, rock and geomaterials, or rather cementitious and granular materials in the parlance of mechanics of materials, are known to exhibit strong coupling between the volumetric and deviatoric behavior. In this case of cementitious particle composites, the Reynolds effect is responsible for inelastic dilatancy which leads to discontinuous failure modes which vary strongly between decohesive cracking in direct tension and mixed-mode shear-compression failure (shear faulting) in simple shear and triaxial compression/extension. Thus in cementitious particle composites, the formation of localized macrodefects is critical for determining the vulnerability of protective structural components and systems.

Deterioration of the mechanical properties of structural materials is a field of critical importance when the life-cycle behavior and the limit state performance of structures are to be evaluated. Thereby, degradation of stiffness and strength is of paramount importance, but even more so, the degradation of ductility in view of the increasing use of high performance structural materials with reduced toughness.

At the core of failure processes in structures is the energy absorption capacity or the overall ductility of the structural material. It is primarily the ductility and the deformation capacity that is greatly affected by localization and the concomitant formation of spatial discontinuities. The appearance of 'weak' discontinuities in the strain field indicates the onset of failure and the degradation of the continuum into a discontinuum in which strong mesh dependency is to be expected in numerical computations. In order to avoid the formation of "weak" discontinuities, a number of researchers proposed to adopt the *Cosserat theory of micropolar continua* to regularize discontinuities. This theory introduces a characteristic length in a natural manner, which distributes the failure process in the vicinity of localization events in the spirit of non-local effects.

The analysis of discontinuities in a Cosserat continua leads to localization conditions that differ from those of classical continua in a basic sense. First, the localization tensor has to be augmented to account for discontinuities of the velocity gradient, and also of the rate of rotation gradient. Second, another condition arises from the moment equilibrium equations, that plays a prominent role in the regularization of the underlying jump condition.

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## Accomplishments

To demonstrate the analogies and differences between classical continua and micropolar Cosserat continua, a thorough derivation of the fundamentals was carried out. The basic concepts were illustrated with the aid of Mohr's circle which is generalized to depict non-symmetric states of stresses and strain. The localization condition for weak discontinuities was examined analytically and reinterpreted geometrically along the line of the Mohr failure envelope. The study of discontinuous failure led to a geometric representation of the critical failure directions that correspond to the first localization condition. If the plastic yield condition is formulated in terms of the symmetrized stress state, the interdependence of the two localization conditions restricts bifurcation to pure mode I failure. The second localization condition is the main reason why mode I failure is the only discontinuous failure mode which can not be regularized by the micropolar formulation.

In the course of the project, localization properties were studied in elastoplastic Cosserat continua at the constitutive and at the structural levels, whereby the localization results of classical elastoplasticity are recovered as a particular case when the Cosserat shear modulus  $G_c$  is zero. An analytical solution of localization in Cosserat continua was developed, and the critical hardening modulus was obtained as a function of the stress state and the material parameters. In order to develop a geometrical solution, the non-symmetric stress state was represented in the Mohr plane of normal and shear stresses. Thereby it is understood that the non-symmetric stress tensor can have real or complex principal values, whereby the maximum normal stress does no longer coincide with the major principal stress, unless the stress tensor is symmetric. For the elastoplastic Cosserat formulation a symmetric Drucker-Prager model was adopted where the stresses and couple stresses are combined in the yield function, and where a single plastic multiplier couples the plastic strains and plastic curvatures.

An analytical localization condition was obtained from traction continuity requirement across the discontinuity surface that could be mapped in the transformed Mohr space of normal and shear stresses when there are no couple stresses. The tangency condition with the Mohr circle did provide critical values of the localization angle and the associated hardening parameter. However, a second condition had to be satisfied when the angular momentum balance was considered on each side of the discontinuity surface. In order to satisfy this condition either the Cosserat shear modulus has to be zero - in that case the results of classical elastoplasticity are recovered - or localization has to occur in the form of mode I decohesive failure. Considering this requirement an analytical expression of the critical hardening parameter was obtained. In short, localization in Cosserat continua can only take place in mode I if a symmetrized yield condition is adopted. Thereby, the resulting failure angle is measured with respect to the maximum stress direction that does not coincide with the direction of major principal stress. Moreover, the critical hardening parameter that satisfies both localization conditions is, in general, lower than the one necessary to satisfy the first condition. As a result, more severe softening is required in Cosserat continua in order to trigger localization as compared to classical continua.

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Continuous and discontinuous failure modes were studied also at the structural level, in order to examine the manifestation of the two localization conditions in Cosserat continua. The numerical studies were performed on a square panel subjected to loading in tension under plane stress. The computational examples showed that classical elastoplastic continua results were recovered if the Cosserat shear modulus was set to zero,  $G_c = 0$ , or if the characteristic length was very small with respect to the size of the computational domain  $\ell_c/L \ll 1.0$ .

If  $G_c \neq 0$ , and if the hardening parameter was larger than the critical one, then the localization condition is not satisfied and discontinuous failure bands did not form. The structural response showed little mesh-dependency until deformation trapping took place in a boundary layer, exhibiting weak regularization and conversion to mode I failure. On the other hand, if the hardening modulus was lower than the critical one, then localized failure was triggered right away in the form of mode I type failure by inherent or induced imperfections which did result in strong mesh dependency and snap-back at the structural level.

## Conclusions

To conclude, micropolar Cosserat continua provide regularization of classical continua formulations in a weak sense since more severe softening was required to satisfy the two localization conditions simultaneously. In addition, the shear failure mode of classical elastoplastic continua is converted to decohesive mode I failure in Cosserat continua when the Drucker-Prager yield function was formulated in terms of the symmetrized stress tensor.

## List of Publications

1. Carol, I. and Willam, K., (1997) "Applications of Analytical Solutions in Elasto-Plasticity to Localization Analysis of Damage Models", Proc. COMPLAS V, D.R.J. Owen, E. Onate and E. Hinton (eds.), Publ. of CIMNE, Barcelona, pp. 714-719.
2. Iordache, M.-M., (1996), "Failure Analysis of Classical and Micropolar Elastoplastic Materials", PhD Dissertation, CEAE-Department, University of Colorado, Boulder, Report CU/SR-96/2.
3. Iordache, M.-M. and Willam, K., (1997) "Mohr-Type Analysis of Failure in Drucker-Prager Materials", Proc. COMPLAS V, D.R.J. Owen, E. Onate and E. Hinton (eds.), Publ. of CIMNE, Barcelona, pp. 1671-1678.
4. Iordache, M.-M.. and Willam, K., (1997), "Localized Failure Modes in Cohesive-Frictional Materials," accepted for publication, Intl. J. Cohesive-Frictional Materials, Vol. 3.
5. Iordache, M.-M.. and Willam, K., (1997), "Localized Failure Analysis in Elastoplastic Cosserat Continua," in press, Comp. Meth. Appl. Mech. Eng., Vol. 149.

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6. Lee, Y.-H. and Willam, K., (1997), "Mechanical Properties of Concrete in Uniaxial Compression," in press, ACI Materials Journal, Vol. 94, No 6.
7. Willam, K. and Iordache, M.-M., (1996a), "Deterioration Measures for Concrete Materials," 60th Anniversary Volume in honor of Josef Eibl, H. Hilsdorf and G. Kobler (eds.), Institut für Baustoffkunde und Massivbau, IfBM, University of Karlsruhe, pp. 183-202.
8. Willam, K. and Iordache, M.-M., (1996b), "Computational Simulation of Discontinuous Failure Processes", MECOM 96, 5th Argentine Congress of Computational Mechanics, Sept. 10-13, 1996, G. Etse and B. Luccioni, eds. Vol. XVII, Mecanica Computacional, AMCA, Universidad Nacional de Tucuman, pp. 227-242.

### Conference Presentations:

1. Willam, K. and Iordache, M.-M., "Computational Simulation of Discontinuous Failure Processes," MECOM 96, Plenary Lecture at 5th Argentine Congress of Computational Mechanics, Tucuman, Argentina, Sept. 10-13, 1996.
2. Willam, K. and Iordache, M.-M., "Localized Failure Analysis in Elastoplastic Cosserat Continua," Advances in Computational Mechanics, TICAM - The University of Texas Austin, Jan. 13-15, 1997.
3. Willam, K., "Challenges of the Simple Shear Problem", Invited Seminar in Applied Mechanics, Technical University of Catalonia, Barcelona, May 16, 1997.
4. Willam, K. and Iordache, M.-M., "Failure Simulations in Elastoplastic Cosserat Continua", 4th National Congress of Computational Mechanics, San Francisco, August 6-8, 1997.